Optimization Model for assessment environmental pollution under restricted input information

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The basic investigation phases and problems:

- 1. Measurement of environmental pollution
- 2. Optimization of observing system
- 3. Analysis of under-torch observing
- 4. Reconstruction of aerosol sedimentation fields
- 5. Definition of sources parameters

2. Optimization of observing system

1) Let experiment was carried out in N-1 point according to a plan εN-1. We find a point $\vec{x}_N$ such, that

$$d\left(\vec{x}_N, \vec{e}_{N-1}, \vec{q}_{N-1}\right) = \max_{x \in \Pi} d\left(\vec{x}, \vec{e}_{N-1}, \vec{q}_{N-1}\right),$$

$$d\left(\vec{x}, \vec{e}_{N-1}, \vec{q}_{N-1}\right) = \nabla^{r} q \cdot M^{-1} \cdot \nabla q \qquad (1)$$

2) In the point $\vec{x}_N$ additional observation is carried out.

3) We find estimations $\vec{q}_N$ by the observation according to the plan.

$$e_n = \frac{N-1}{N} \cdot e_{N-1} + \frac{1}{N} \cdot e\left(\vec{x}_N\right), \qquad (2)$$

3. Analysis of under-torch observing

$$u(z)\frac{\partial q}{\partial x} - w\frac{\partial q}{\partial z} = \frac{\partial}{\partial z}k(z)\frac{\partial q}{\partial z} + \frac{\partial}{\partial y}v(z)\frac{\partial q}{\partial y}\,, \qquad \textbf{(3)}$$

$$k\frac{\partial q}{\partial z}\bigg|_{z=0} = 0\,, \quad q\big|_{|\vec{x}|\to\infty} \to 0\,, \quad q\big|_{x=0} = Md(y)d(z-H)\,, \qquad \textbf{(4)}$$

$$u(z) = u_1\left(\frac{z}{z_1}\right)^n, \qquad k(z) = k_1\left(\frac{z}{z_1}\right)^m, \qquad v(z) = k_0 u(z) \qquad \textbf{(5)}$$

$$q(\vec{x},\vec{q}) = \frac{q_1}{x^{3/2}} \exp\left(-\frac{q_2}{x} - \frac{q_3 y^2}{x} \right). \qquad (6)$$

$$q_1 = e^{3/2} \cdot q_{\max} \, x_{\max}^{3/2}, \quad q_2 = \frac{3}{2} x_{\max}, \quad q_3 = \frac{1}{4k_0},$$

$$q_w(\vec{x},\vec{q}) = \frac{q_1}{x^{3/2}} \exp\left(-\frac{q_2}{x} - \frac{q_3 y^2}{x} \right) \sum_{i=1}^{K} \frac{p_i \, q_2^{q_4 w_i}}{\Gamma(1 + w_i q_4) \, x^{q_4 w_i}} \qquad (7)$$

$$q_4 = \frac{1}{k_1 (1+n)}.$$

$$r_k = q\left(\vec{x}_k, \vec{q}\right) + x_k \ , \qquad \textbf{(8)}$$

$$E\left[x_n\right] = 0 \ , \quad E\left[x_k x_j\right] = d_{kj} s_k^2 \ , \quad k, j = \overline{1, N} \ .$$

$$J_N\left(\vec{q}\right) = \sum_{k=1}^{N} s_k^{-2} \left[r_k - q\left(\vec{x}_k, \vec{q}\right)\right]^2 \ . \qquad \textbf{(9)}$$

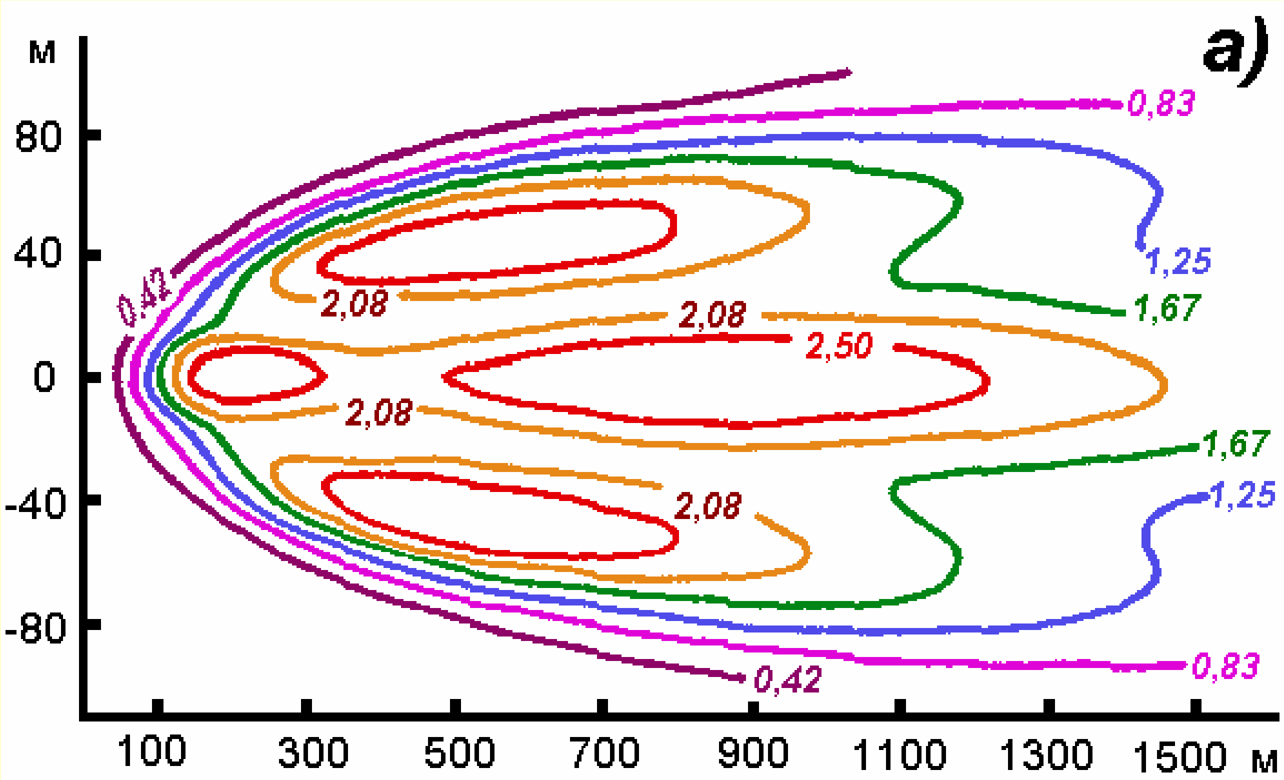


Fig. 1a. ***Surface concentration Dispersion of an light impurity concentration for*** *x_{max}= 600 m,* *k_0 = 0,8 m*

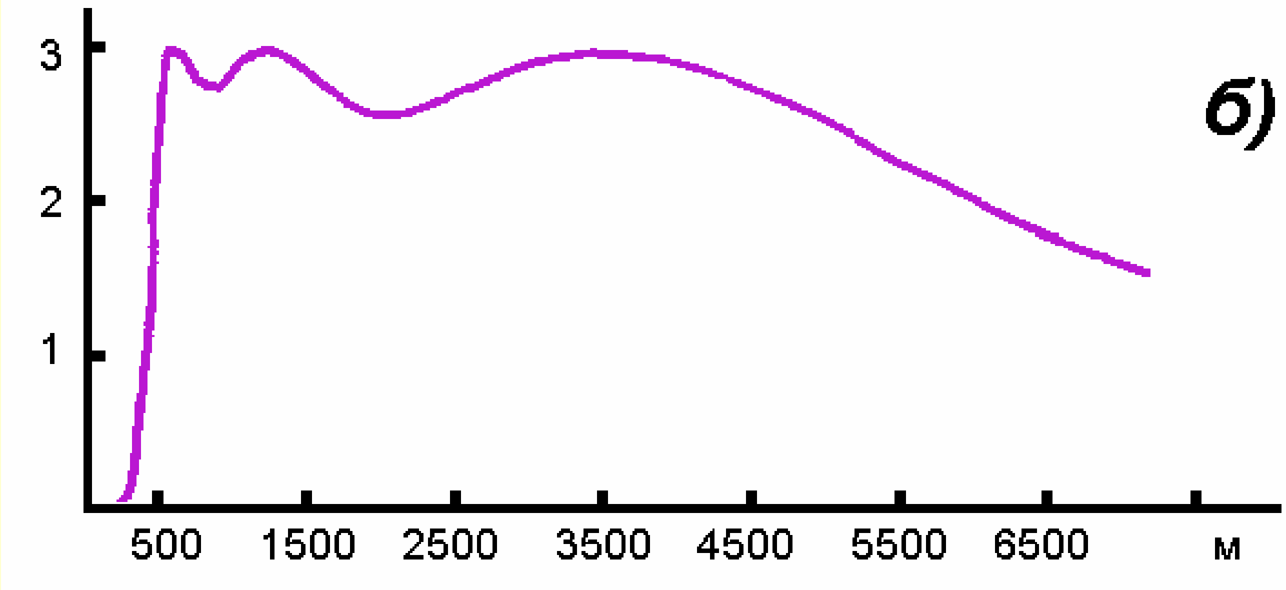


Fig. 1b. ***Axial Dispersion of a concentration field of a heavy impurity for*** *x_{max}= 1300 m,* *w = 20 sm/**c***

4. Reconstruction of aerosol sedimentation fields

$$\overline{q}_{\overline{t}} = \int_0^\infty q\, r_{t,\overline{t}}(q)\, dq \qquad (10)$$

4.1. Aerosol pollution of local scale

a). Point source

$$\overline{q}(r, j) = \qquad (11)$$

$$= \iint_\Omega q(r, j, K_1, u_1)\, P_1(K_1, u_1)\, dK_1 du_1$$

$$P_1(K_1,u_1)=p'(u_1)\,p''(l), \qquad l=\frac{k_1}{u_1}, \qquad (12)$$

$$p''(l)=d\left(l-\bar{l}\right), \qquad p''(l)=\frac{a^{K-1}\,l^{-K}}{\Gamma(K-1)}e^{\frac{-a}{K}} \qquad (13)$$

$$\bar{q}(r,j)=$$

$$=\frac{QP(j+180^\circ)}{\sqrt{2p}\,j_0 r^2}\cdot\iint_{\Omega_1}\frac{1}{n+1}e^{\frac{-H^{n+1}}{l(1+n)^2 r}}\,l\,p'(u_1)\,p''(l)\,dl\,du_1=$$

$$=\frac{QP(j+180^\circ)\,\bar{l}}{\sqrt{2p}\,(1+n)\,j_0 r^2}e^{\frac{-H^{n+1}}{\bar{l}(1+n)^2 r}}\cdot\int_0^u p'(u_1)\,du_1= \qquad (14)$$

$$=q_1\frac{P(j+180^\circ)}{r^2}e^{\frac{-q_2}{r}}$$

$$q_1 = \frac{Q\bar{l}}{\sqrt{2p}\,(1+n)\,j_0}\int_0^u p'(u_1)\,du_1\,, \qquad q_2 = \frac{H^{1+n}}{\bar{l}\,(1+n)^2} \quad (15)$$

Monodisperse case

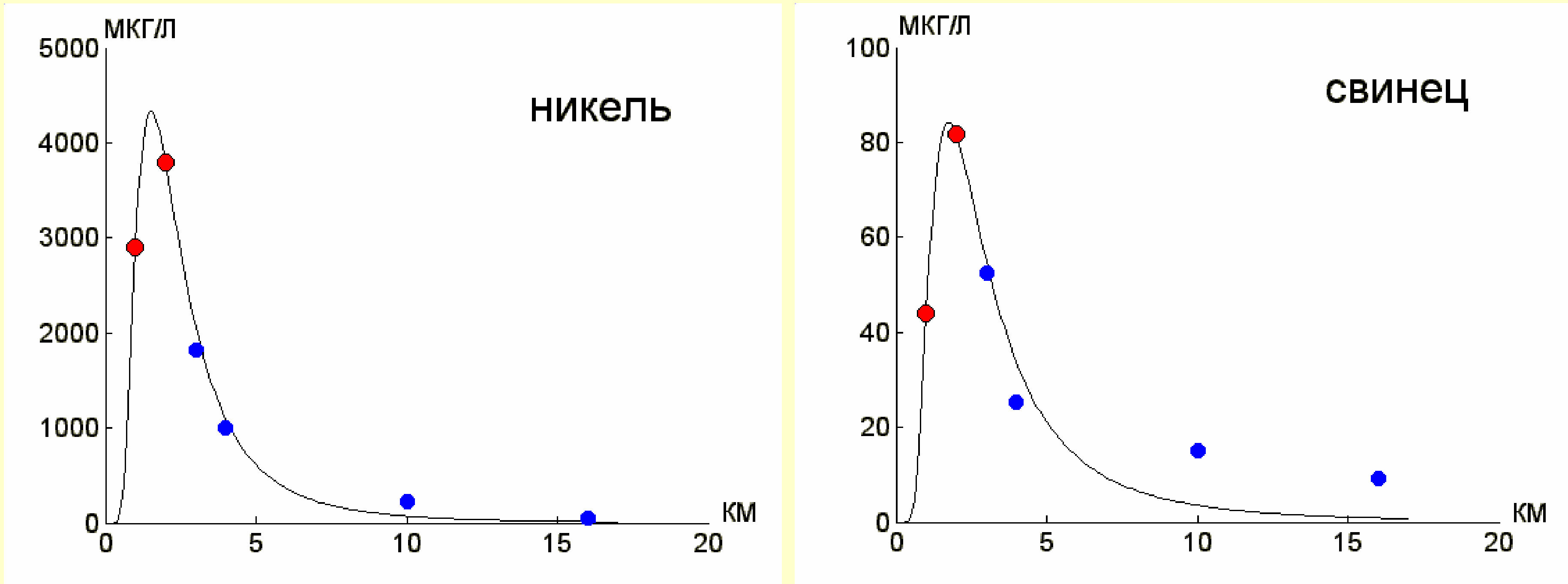
$$q_w(r,j) = q_{1w}\cdot P(j+180^\circ)\cdot r^{q_{3w}}\cdot e^{\frac{-q_2}{r}} \quad (16)$$

$$q_{1w} = \frac{QH^{(1+n)w_2}}{\sqrt{2p}\,j_0(1+n)^{2w_2+1}\cdot\bar{l}^{\,w_2-1}\cdot\Gamma(1+w_2)}\,, \quad (17)$$

$$q_{3w} = -2-w_2\,, \qquad w_2 = \frac{w}{(1+n)\,\bar{l}_1\,\bar{u}_1}$$

Norilsk brass works

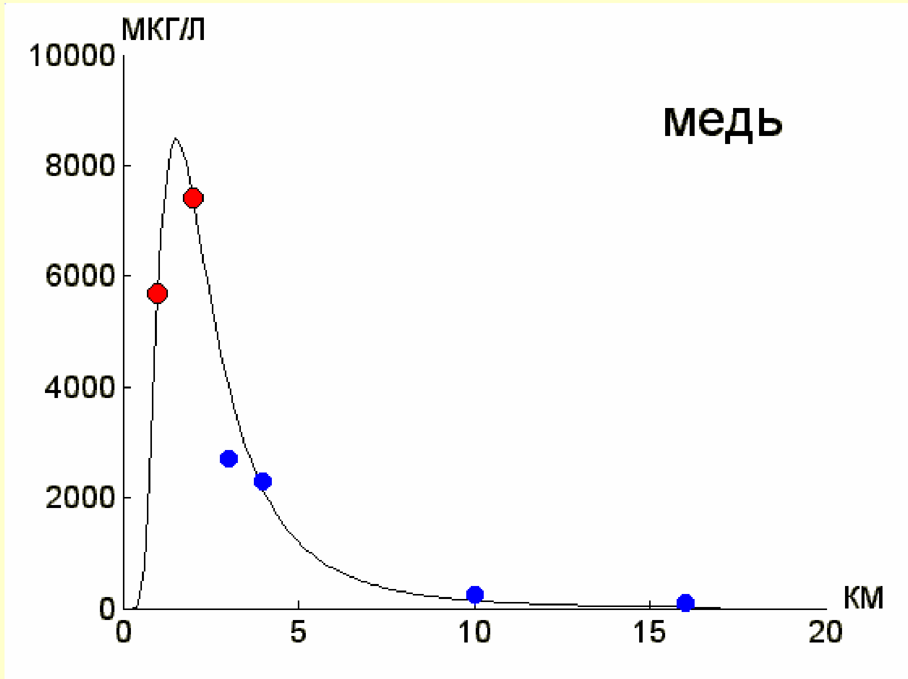
Fig. 2. The content of heavy metals in a firm deposit of snow water in a northeast direction from Norilsk brass works. Restored concentrations of **nickel** and **lead** according local model.

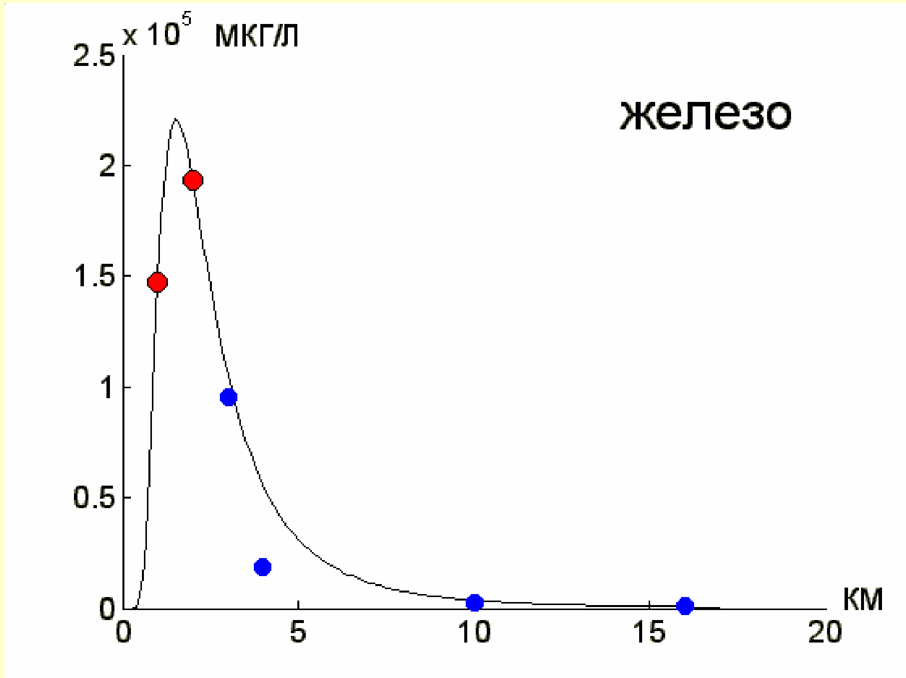


~ - reference point, ~ - control point of observations,

—— - calculated concentration.

Fig. 3. The content of heavy metals in a firm deposit of snow water in a northeast direction from Norilsk brass works. Restored concentrations **of copper** and **iron** according local model.





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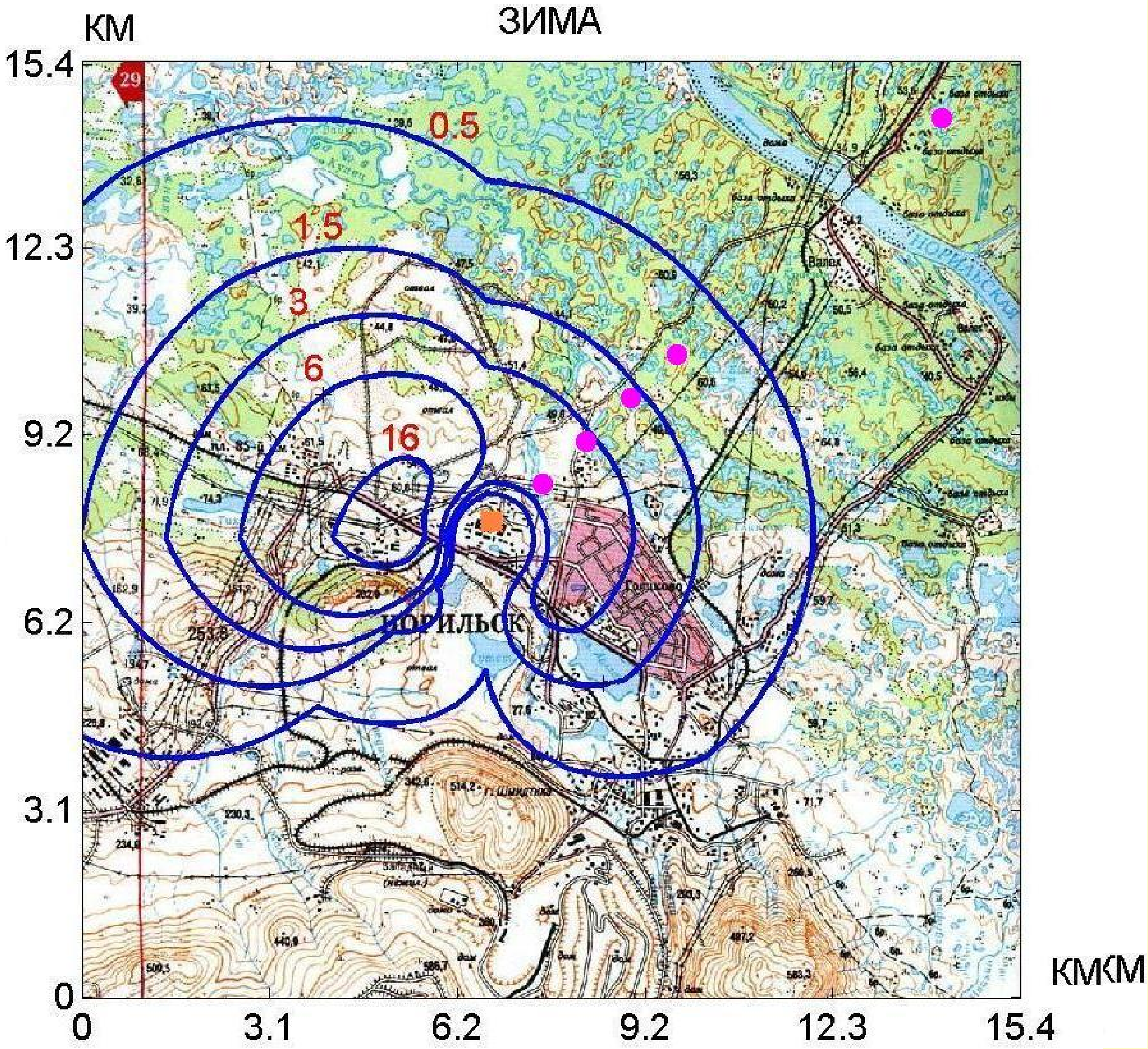
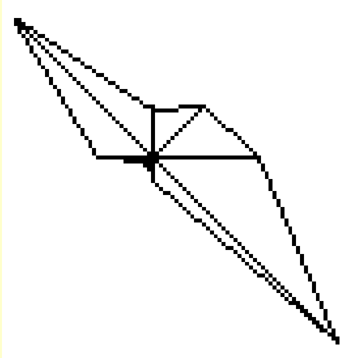


Fig. 4.
Reconstructed aerosol sedimentation fields of **nickels** from brass works **for winter period** (mg/l).



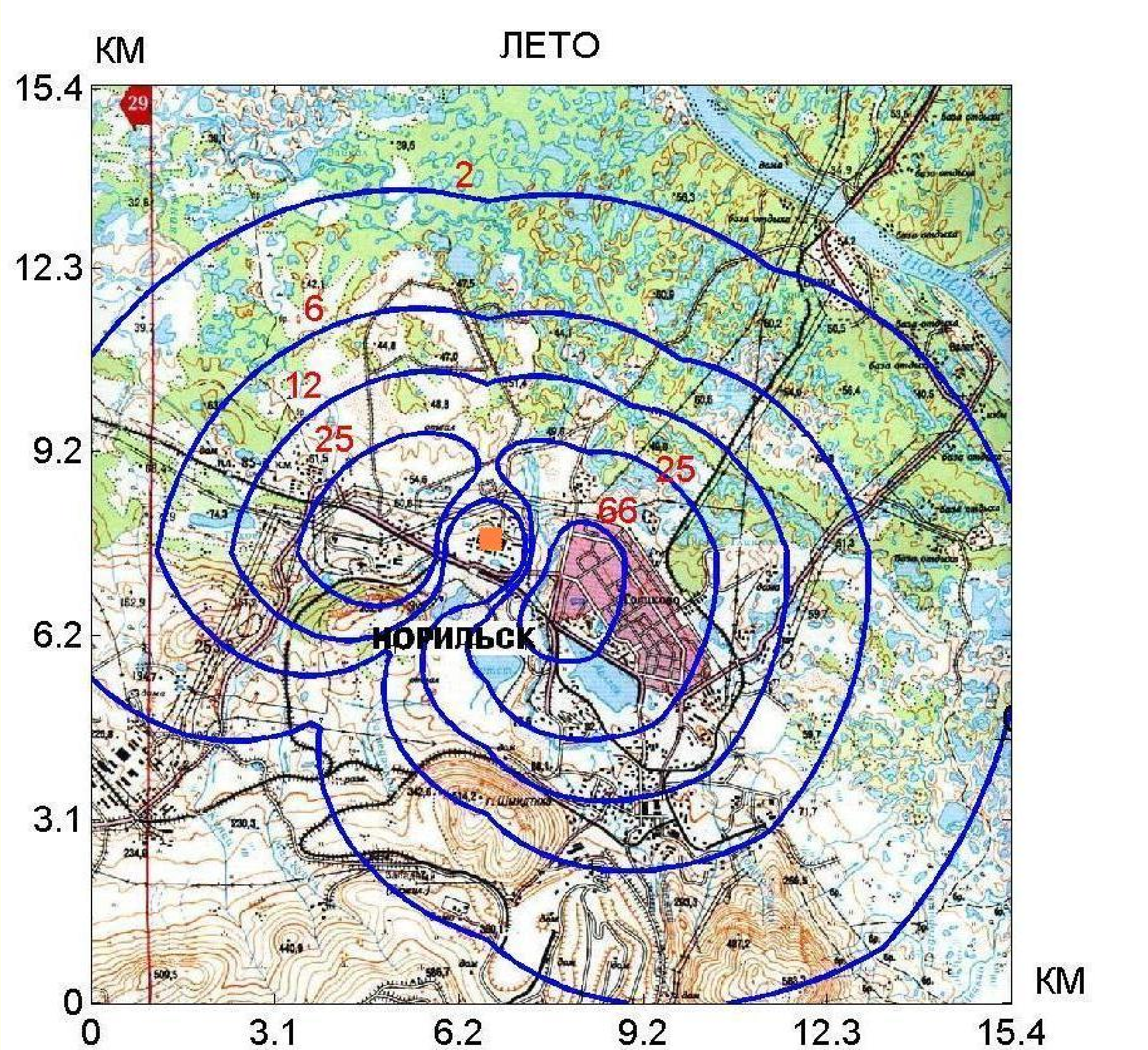


Fig. 5.

Reconstructed aerosol sedimentation fields of **nickels** from brass works **for summer period** (mg/l).

(in % wise upper bound).

- emission source point of snow

sampling

б). Line source (motoway)

$$q(x,y) = \int_{0}^{2p} \int_{L_1}^{L_2} \frac{S(a)}{2\sqrt{p K_0 a}} \cdot e^{-\frac{b^2}{4K_0 a}} \cdot P\left(j + 180^{\mathbf{0}}\right) dh\, dj \quad , \qquad (18)$$

$a = x \cos j + (y - h) \sin j$,

$b = - x \sin j + (y - h) \cos j$,

S(a) – surface concentration from line source

Fig. 6. Plan of snow sampling route.
~ - reference point, ~ - control point of observations

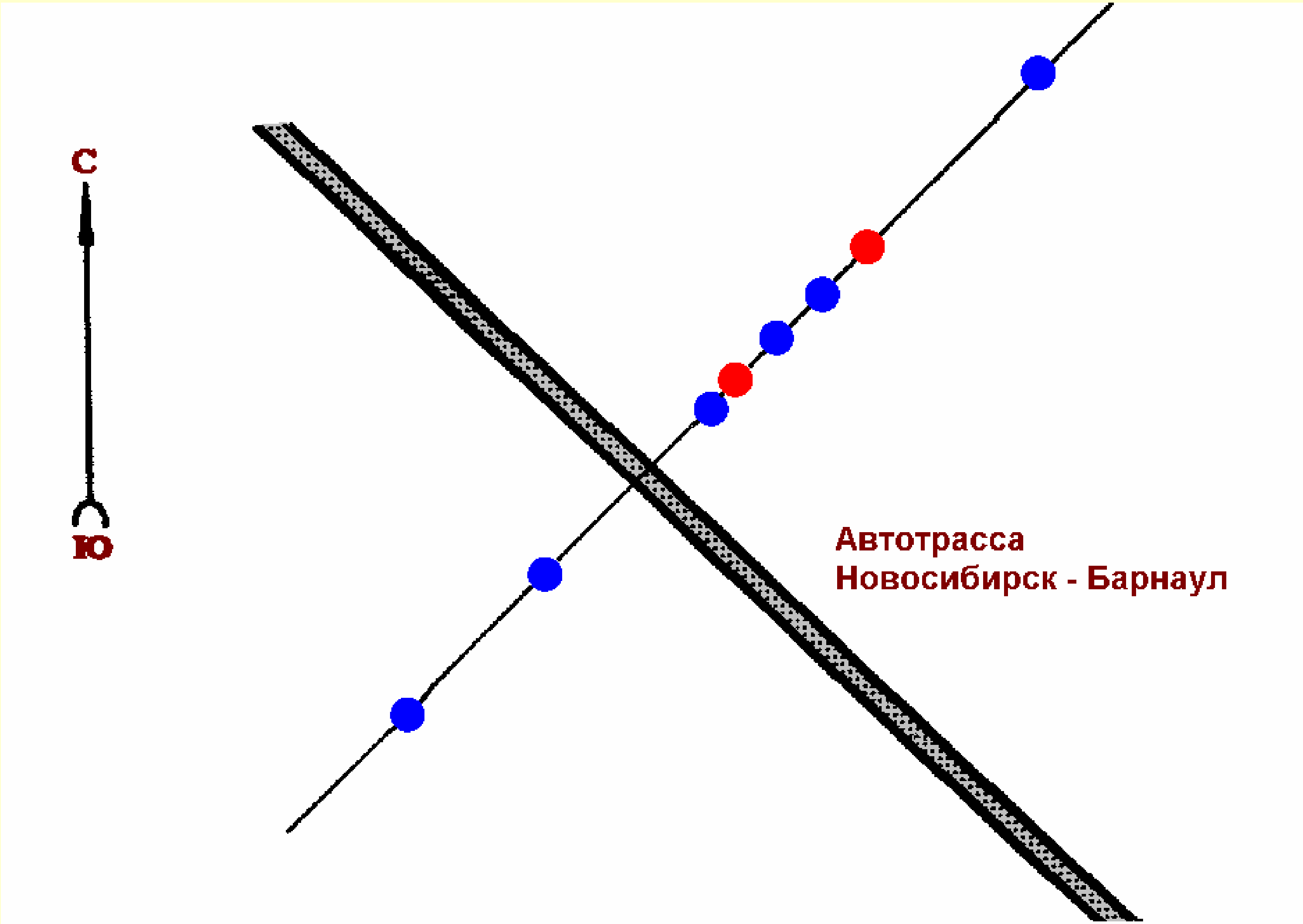


Fig. 7. Specific content of lead in large dyspersated parts (a) and summary content in и fine-dyspersated and water-soluble parts (b).

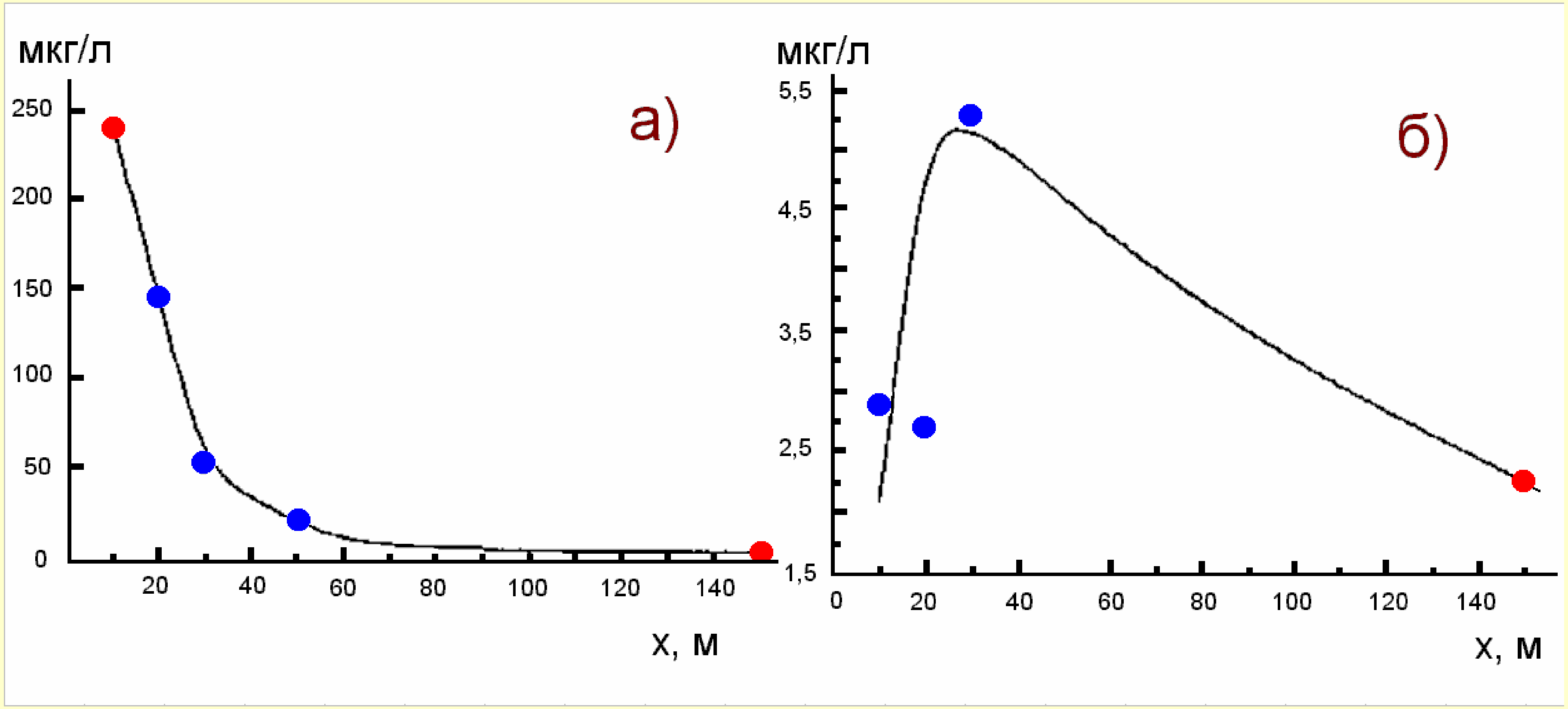


Fig. 8. Fractional Distribution of lead at 50 m distance from motorway.

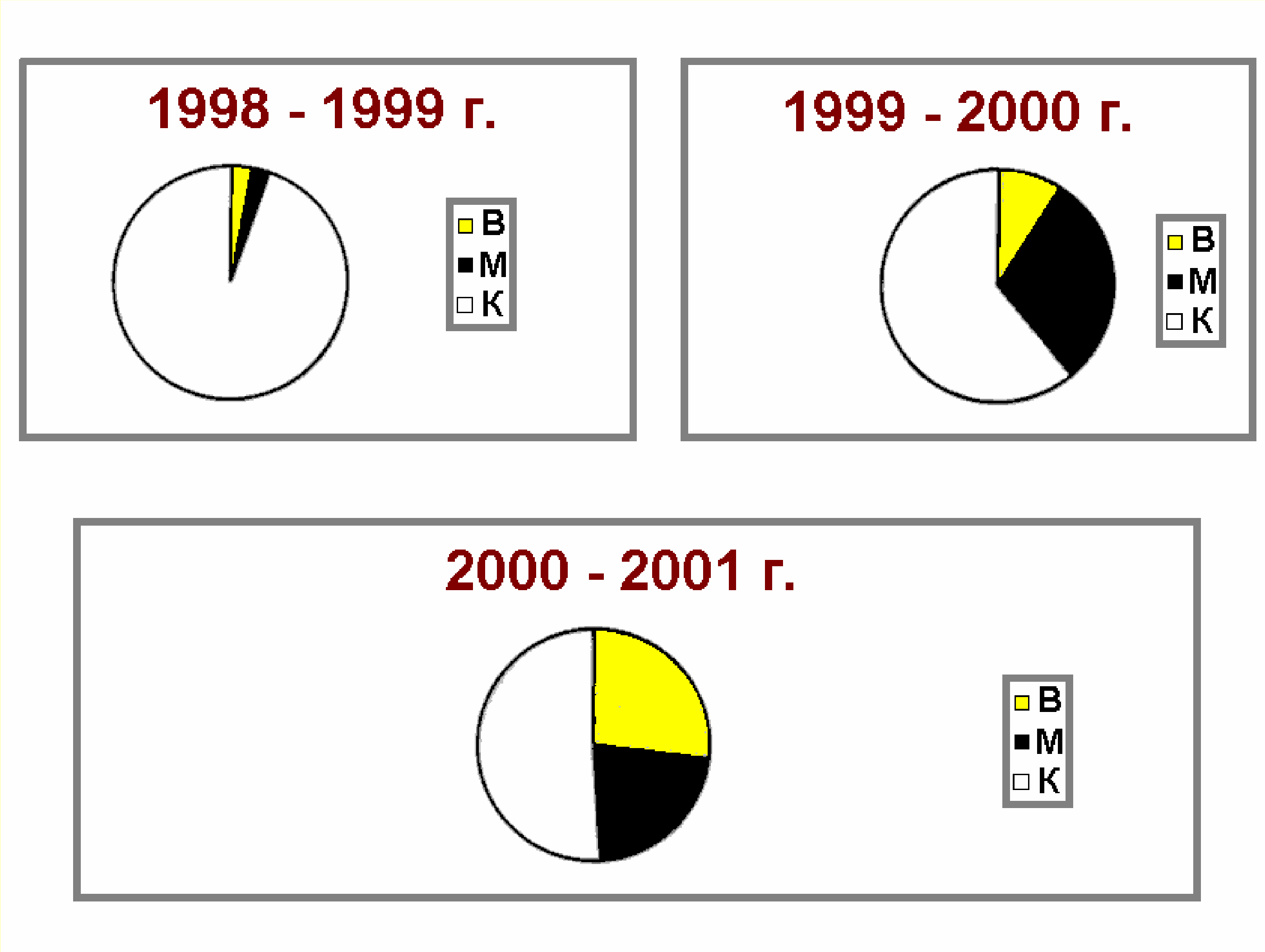
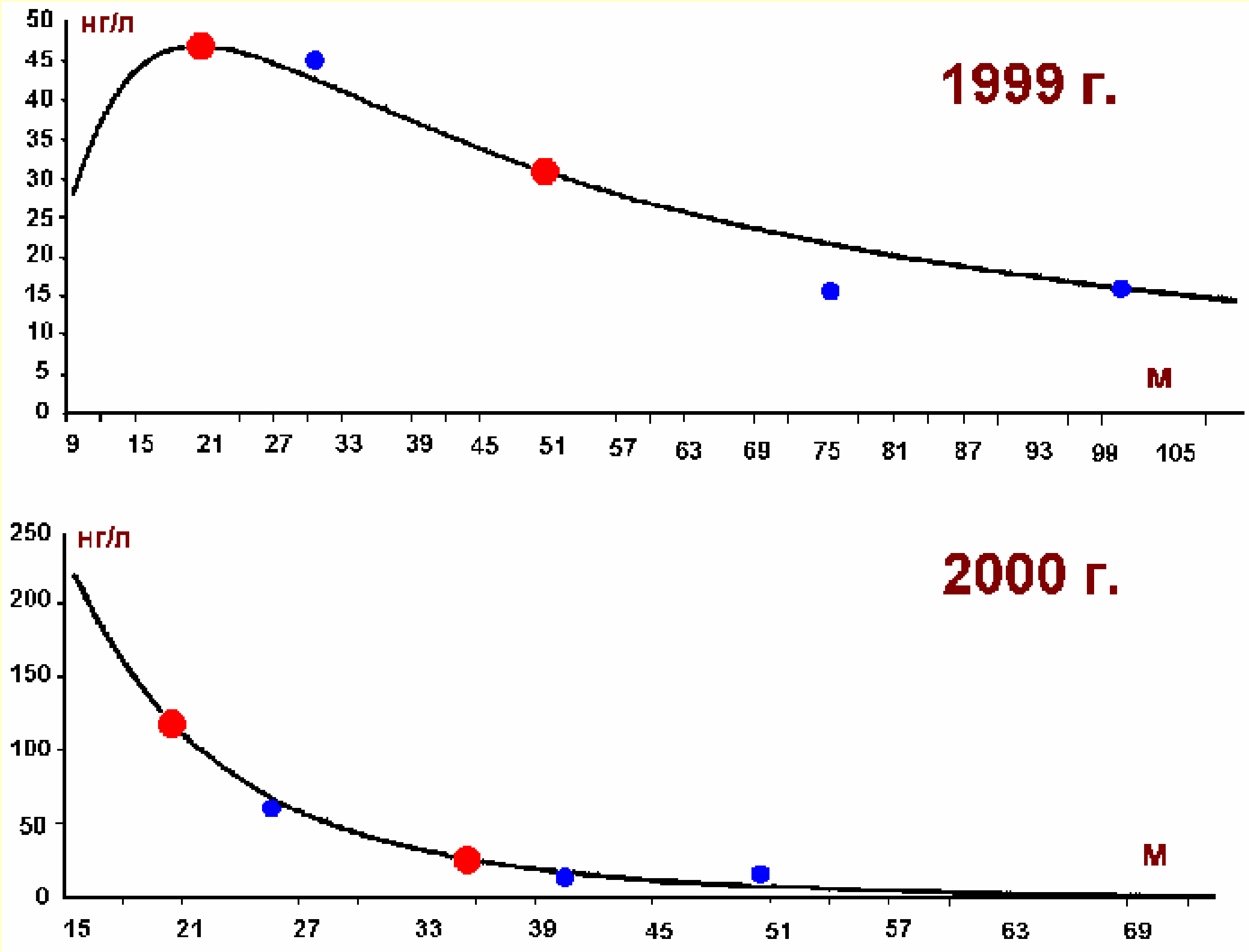


Fig. 9. Calculated and measured Specific content of benzpyrene in snow at the end of winter 1999 и 2000.

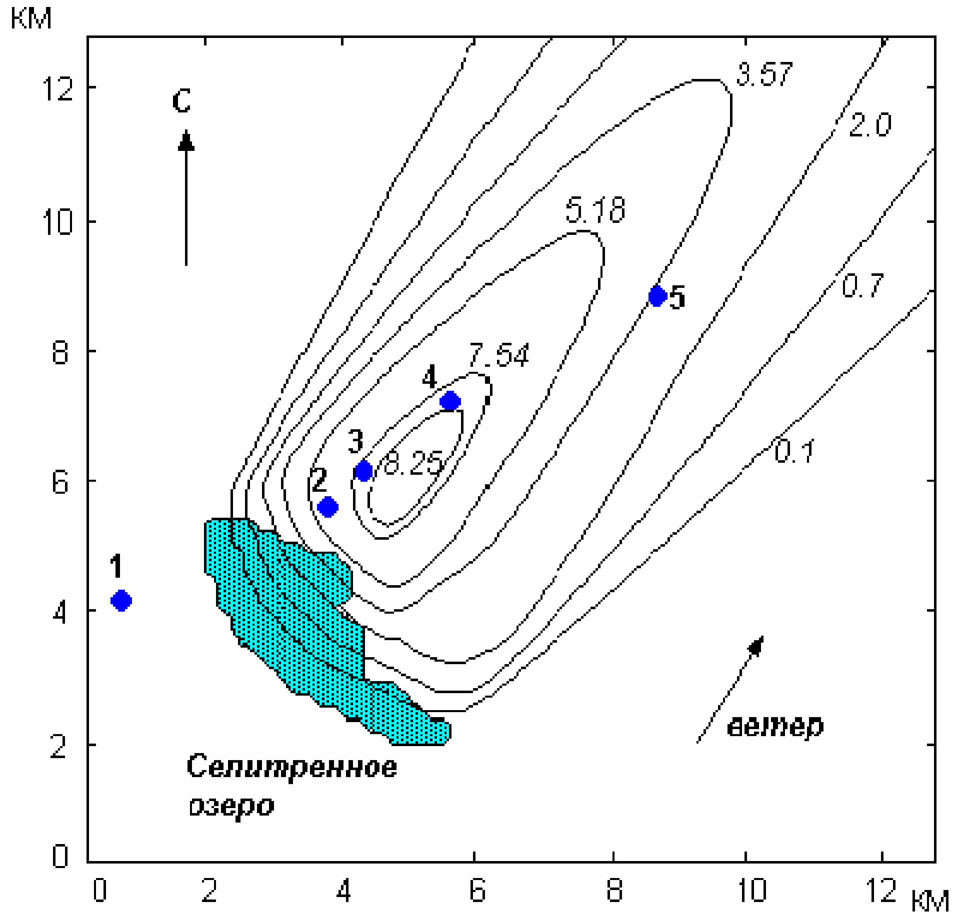


Tabl. 1. Summary estimations of PAC (polynuclear aromatic carbohydrates)

PAC	Summary estimation, *M*, gram/km	
	1999	2000
Bens(a)piren	0,16	0,55
Fluaranten	1,2	1,9
Piren	0,6	1,5

Area source

Fig. 10. Recontracted numerable concentration field of 0,3 - 0,4 mkm fraction of sulfate aerosol in neighborhood of Selitrennoe Lake



№ точки	Измеренные значения счётной концентрации, тыс. частиц
1	1,16
2	5,25
3	7,89
4	8,25
5	3,88

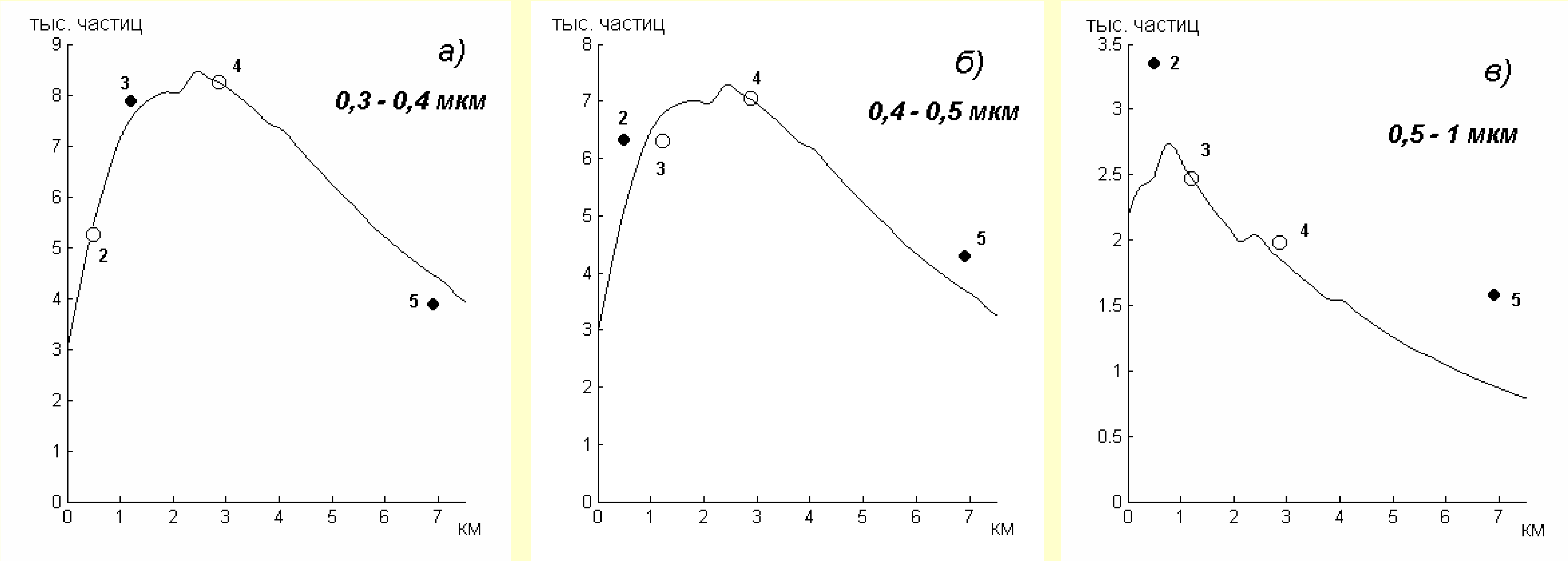
Fig. 11. Measured and calculated means of numerable concentrations for fine fractions of sulfate aerosol

Distance from the lake along observation route is pointed on x axis.

—— - calculated curve,

○ – mesurement at reference points,

● – mesurement at contral points.



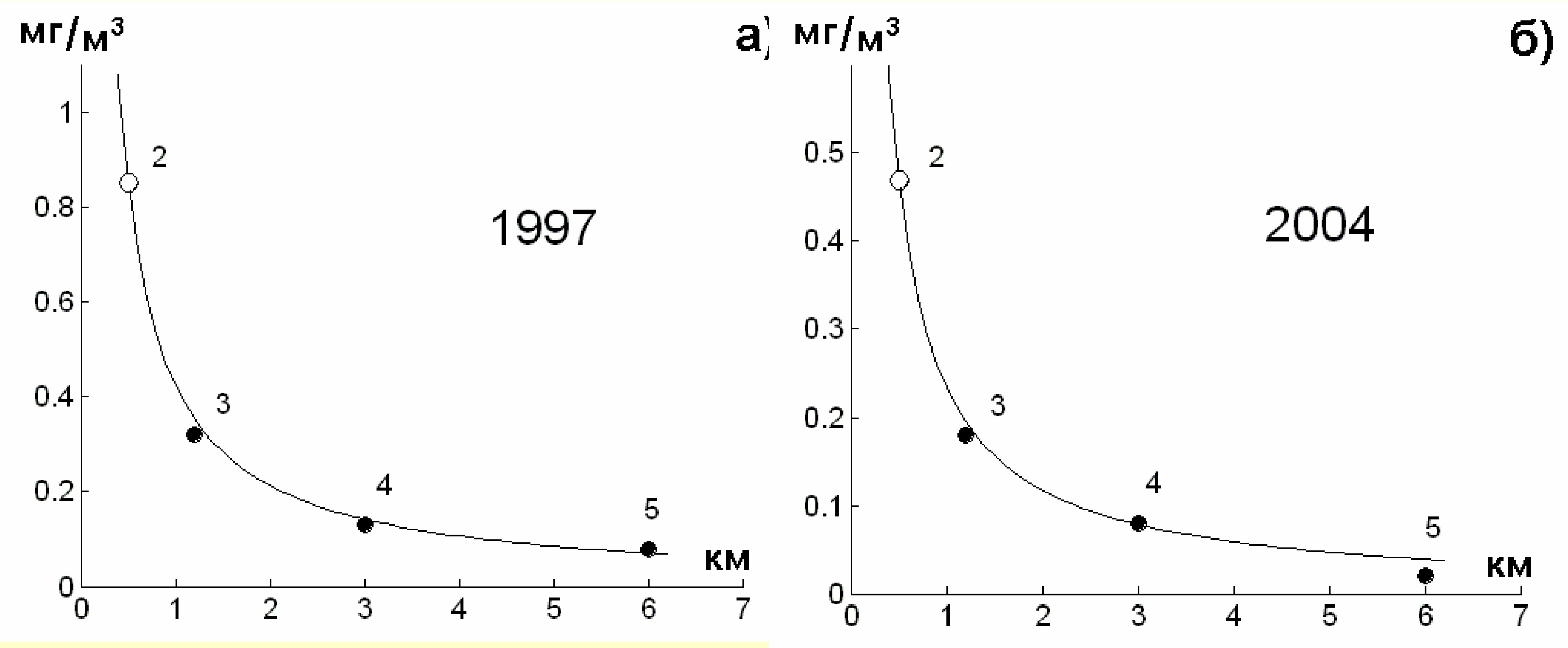


Fig. 12. Mass concentration of sulfate aerosol

b) Regional pollution

$$Q(x,y)=\frac{1}{2p\,uH}\iint\limits_{S}\frac{m(x,h)\cdot P\left(arctg\,\dfrac{y-h}{x-x}+180^{\circ}\right)}{\sqrt{(x-x)^{2}+(y-h)^{2}}}\,dx\,dh \qquad (19)$$

Method of asymptote decomposition

$$Q_1(x,y)=\frac{c}{r}\iint\limits_{S}m(x,h)\cdot$$

$$\cdot\left\{z(j_0)+\left(\frac{p}{2}-1\right)z'(j_0)-z'(j_0)\left(\frac{x}{r^2}x+\frac{y}{r^2}h\right)\right\}dx\,dh= \qquad (20)$$

$$=q_1\frac{z(j_0)+\left(\dfrac{p}{2}-1\right)z'(j_0)}{r}+q_2\frac{z'(j_0)x}{r^3}+q_3\frac{z'(j_0)y}{r^3}$$

Fig. 13. Route Plan of snow mapping in neighborhood of Irkutsk

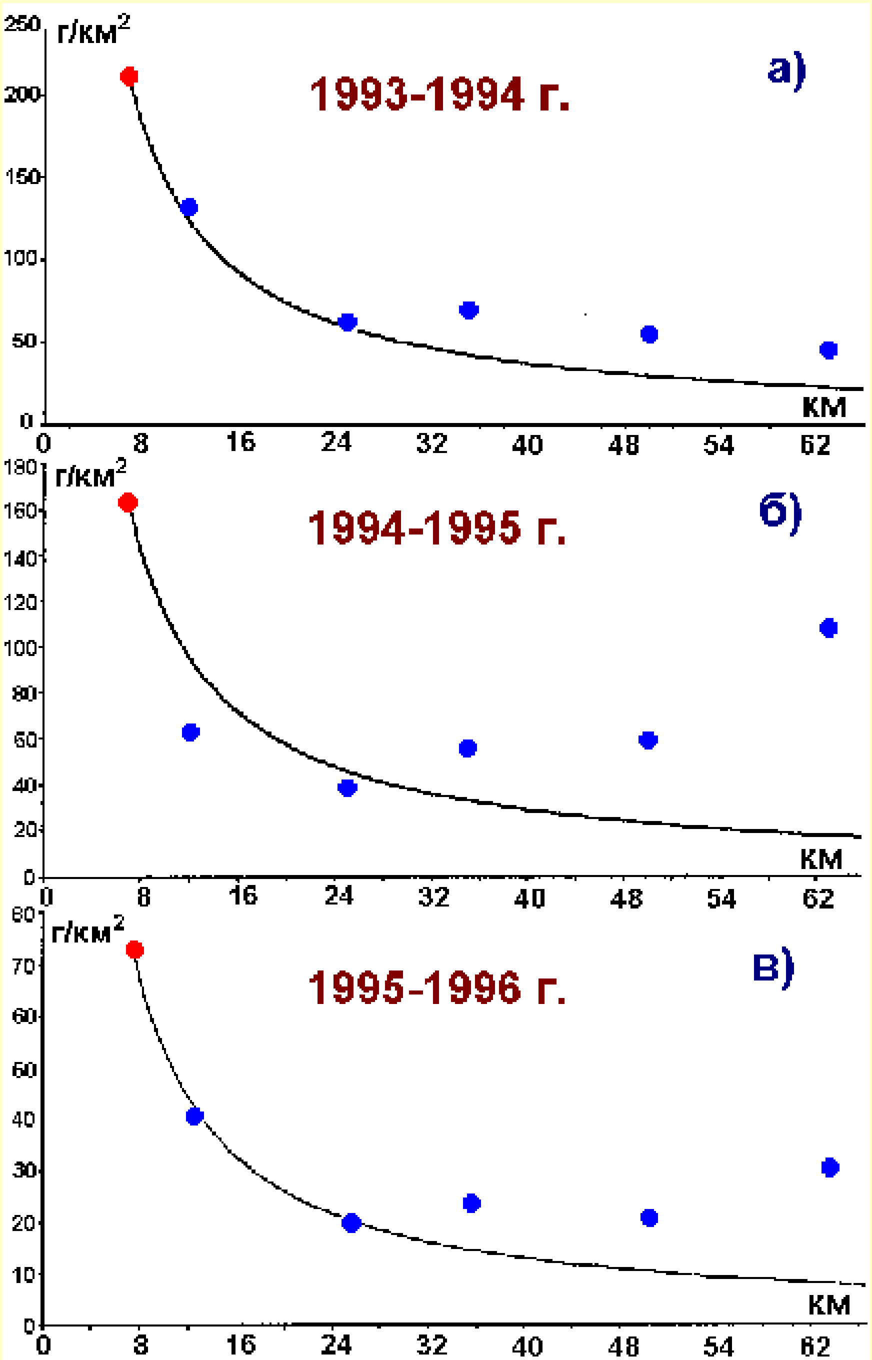


Fig. 14. Sedimentation levels of **beryllium** along Irkutsk-Listvyanka rout over winter period 1993-1994, 1994-1995, 1995-1996

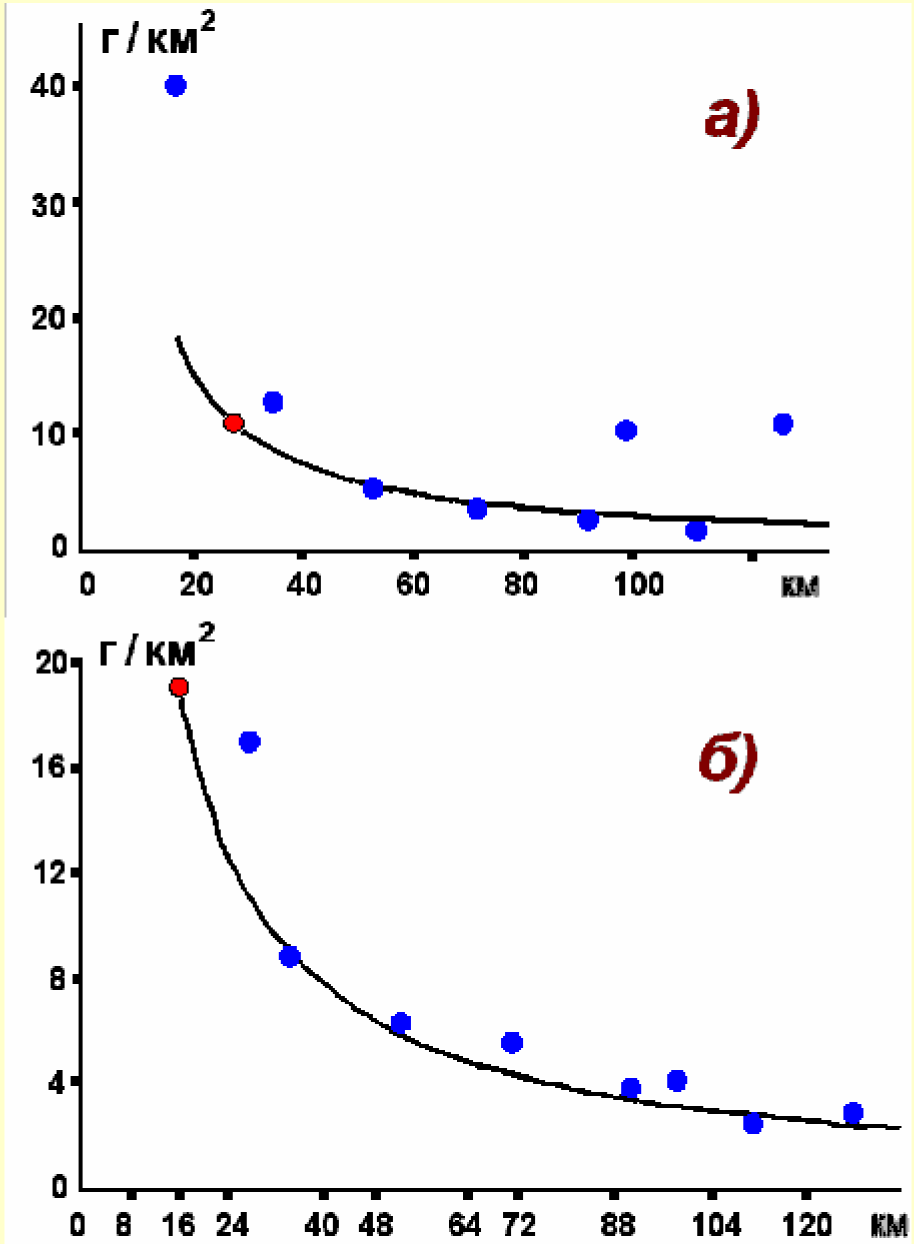


Fig. 15. Sedimentation levels of beryllium along Irkutsk-Bayanday rout over winter period 1994-1995 (**a**), 1995-1996 (b)

High instant source

$$q_w\Big|_{y=z=0} = \frac{2M \cdot k_z}{\sqrt{p}\, s_1 u H^2 k_y} e^{-s^2}\left[1 - s_2\sqrt{s_1}\, r(s)\right] \quad \textbf{(21)}$$

$$s = \frac{1}{\sqrt{s_1}} + s_2\sqrt{s_1}$$

$$s_1 = \frac{4k_z x}{uH^2}$$

$$s_2 = \frac{wH}{4k_z}$$

$$r(s) = e^{s^2} erf(s)$$

$$q_1 = \frac{M}{2\sqrt{p}\,k_y} \qquad q_2 = \frac{2}{H}\sqrt{\frac{k_z}{u}}$$

$$q_3 = \frac{wH}{4k_z} \qquad w(x, q_2, q_3) = \frac{1}{q_2\sqrt{x}} + q_2 \cdot q_3\sqrt{x}$$

$$f(x, \vec{q}) = \frac{q_1}{x} e^{-w^2}\left[1 - q_2 q_3 \sqrt{x}\, r(w)\right] \quad (22)$$

Map of East Ural nuclear trace

a) near zone (less than 30 km)

$$p_1(x, q_1, q_2) \approx \frac{C}{x} \cdot e^{-\frac{w^2}{4k_z u}x} \cdot \int_0^h f(H)\,dH = \frac{q_1}{x} e^{-q_2 x} \qquad \textbf{(23)}$$

$$q_1 = C \cdot \int_0^h f(H)\,dH \qquad\qquad q_2 = \frac{w^2}{4k_z u}$$

$$f(H) = \frac{M(H)}{2\sqrt{p}\,k_y} \cdot e^{-2wH}$$

h - high bound of pollution cloud

C – interaction coefficient

б) Far zone pollution

$$p_2(x) = C\int_0^h M(H)\,q(x,H)\,c_w(x,H)\,dH \quad \textbf{(24)}$$

$$q(x,H) = q_{\max}\exp\left[\frac{3}{2}\left(1-\frac{x_{\max}}{x}\right)\right]\left(\frac{x_{\max}}{x}\right)^{\frac{3}{2}}$$

$$c_w(x,H) = \left(\frac{1.5x_{\max}}{x}\right)^r \qquad r = \frac{w}{k_1(1+n)}$$

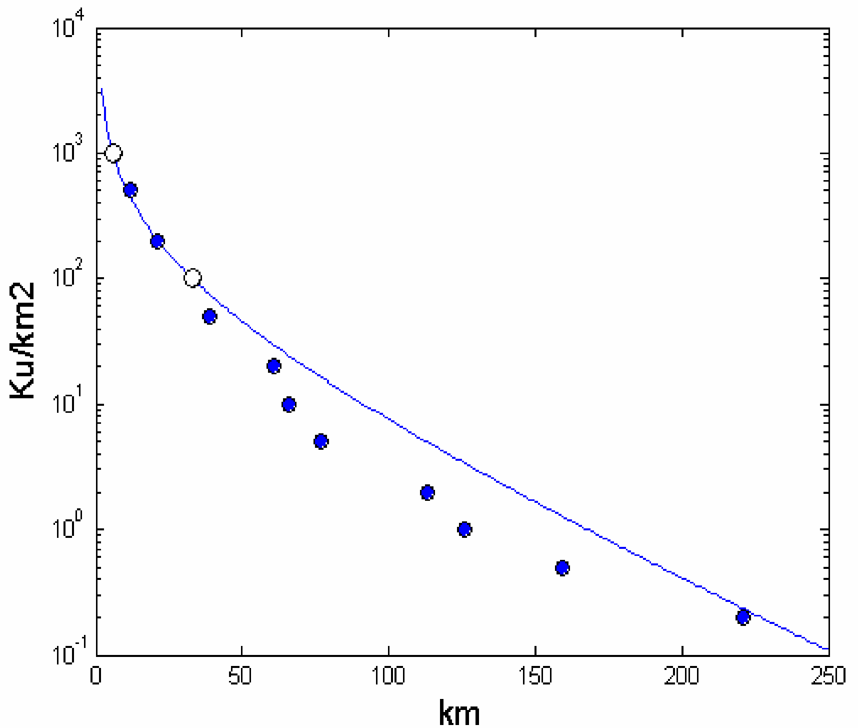
$$\exp(-1.5\,x_{\max}/x) \to 1 \qquad x \to \infty$$

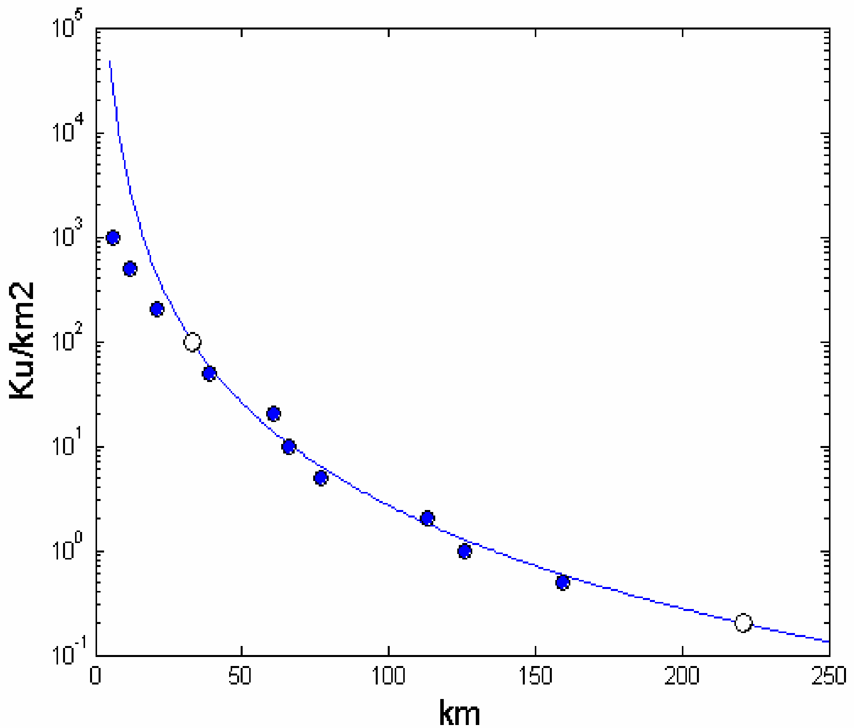
$$p_2(x) \approx \frac{q_1}{x^{1.5+q_2}} \qquad (25)$$

$$q_1 = 1.5^r\, C \int_0^h M(H)\, q_{\max}(H)\, \exp\left[1.5\left(1 - x_{\max}(H)/x\right]\, x_{\max}^{1.5+r}(H)\, dH$$

$$q_2 = r$$

Fig. 16. Reconstructed density of PH sedimentation along axis according BYPC data (1957)



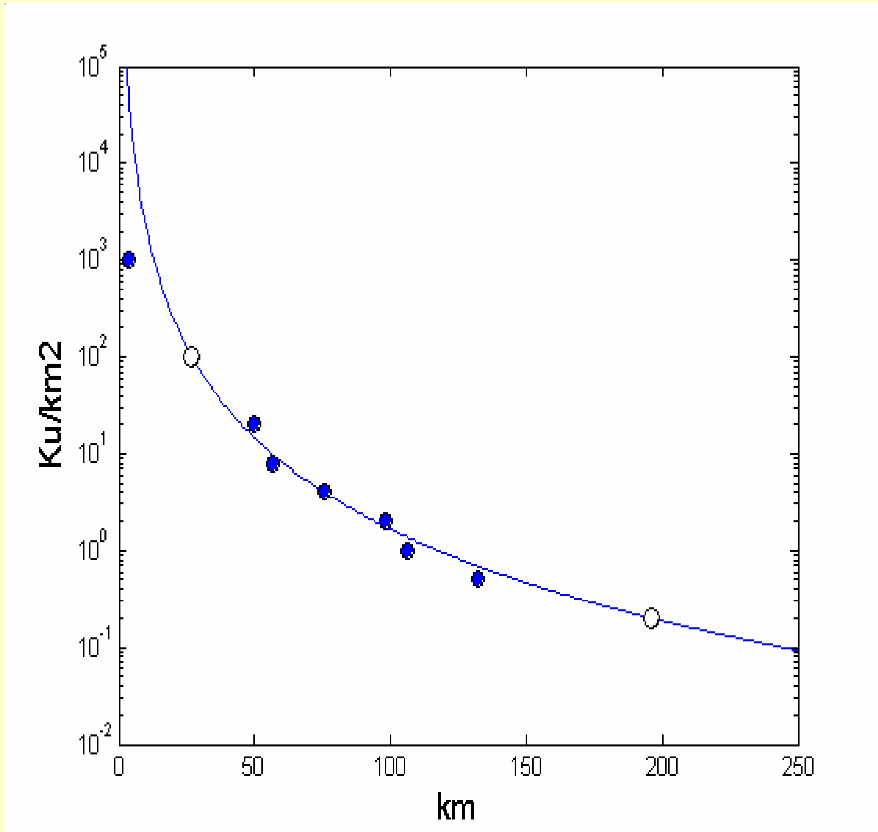


$$k = 1/n \cdot \sum_{j=1}^{n} c_j / p(r_j, \vec{q})$$

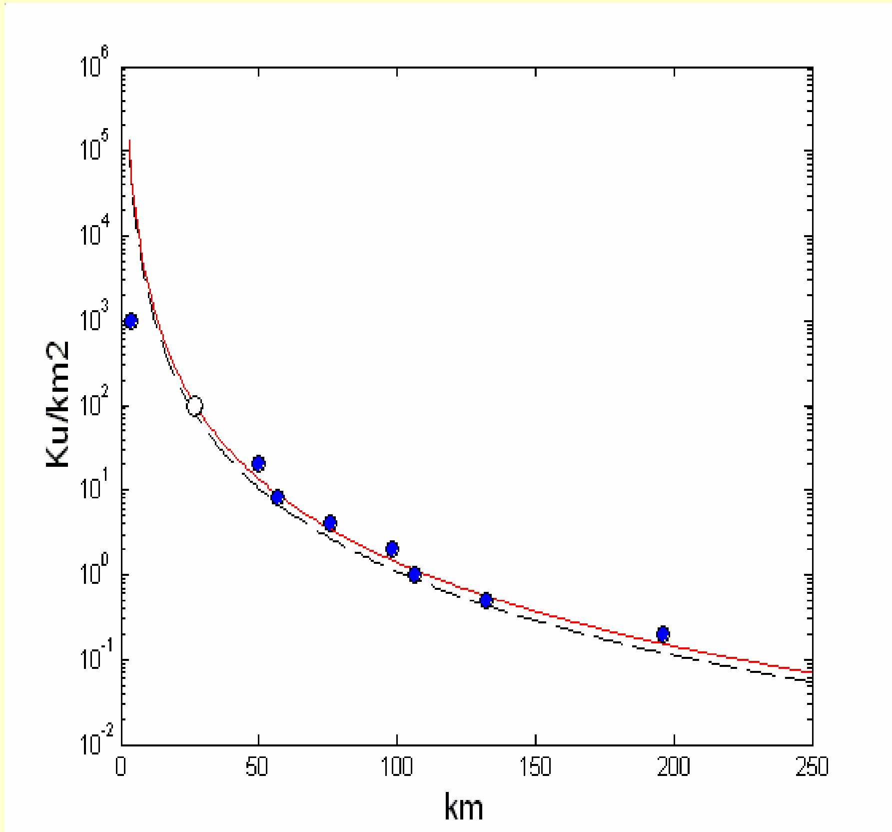
k1=1.06, n=2

k2=0.98, *n*=7 **(26)**

Fig. 17. Reconstructed density of PH sedimentation along axis according ВУРС data (1997)



k3=0.98, *n*=6.



k4=1.45, n=8

k5=1.15, *n*=7

5. Estimation of summary pollution emission

Task 1

$$R(\vec{q}) = \sum_{m=1}^{M} q_m \rightarrow \max_{\vec{q} \in \Omega} \qquad (27)$$

$$q(\vec{x}_n, t, \vec{q}) \le r_n, \quad n = \overline{1, N}. \qquad (28)$$

$$\Omega = \left\{ q_m : 0 \le A_m \le q_m \le B_m, \quad m = \overline{1, M} \right\},$$

Task 2

$$R(\vec{q}) = \sum_{m=1}^{M} q_m \rightarrow \min_{\vec{q} \in \Omega} \qquad (29)$$

$$q(\vec{x}_n, t, \vec{q}) \ge y_n, \quad n = \overline{1, N}.$$

$$q\left(\overset{\mathbf{r}}{x},t\right)=c\left(x,z,t\right)\frac{1}{\sqrt{2p}\,s_{y}}e^{-y^{2}/2s_{y}^{2}}. \qquad \textbf{(30)}$$

$$\frac{\partial c}{\partial t}+\frac{\partial uc}{\partial x}-\frac{\partial}{\partial z}k_{z}\frac{\partial c}{\partial z}=j\left(x,z\right)$$

$$\frac{\partial u}{\partial t}=-\frac{\partial}{\partial z}\overline{u'w'}+fv,\quad \frac{\partial q}{\partial t}=-\frac{\partial}{\partial z}\overline{q'w'}+e_{r}+e_{f},$$

$$\frac{\partial v}{\partial t}=-\frac{\partial}{\partial z}\overline{v'w'}-fu,\quad \frac{\partial q}{\partial t}=-\frac{\partial}{\partial z}\overline{q'w'}-e_{c}+e_{l}, \qquad \textbf{(31)}$$

$$\frac{\partial p}{\partial z}=-gr,\qquad q=\left(\frac{p_{0}}{p}\right)^{g},\quad p=rRT\left(1+0.61q\right),$$

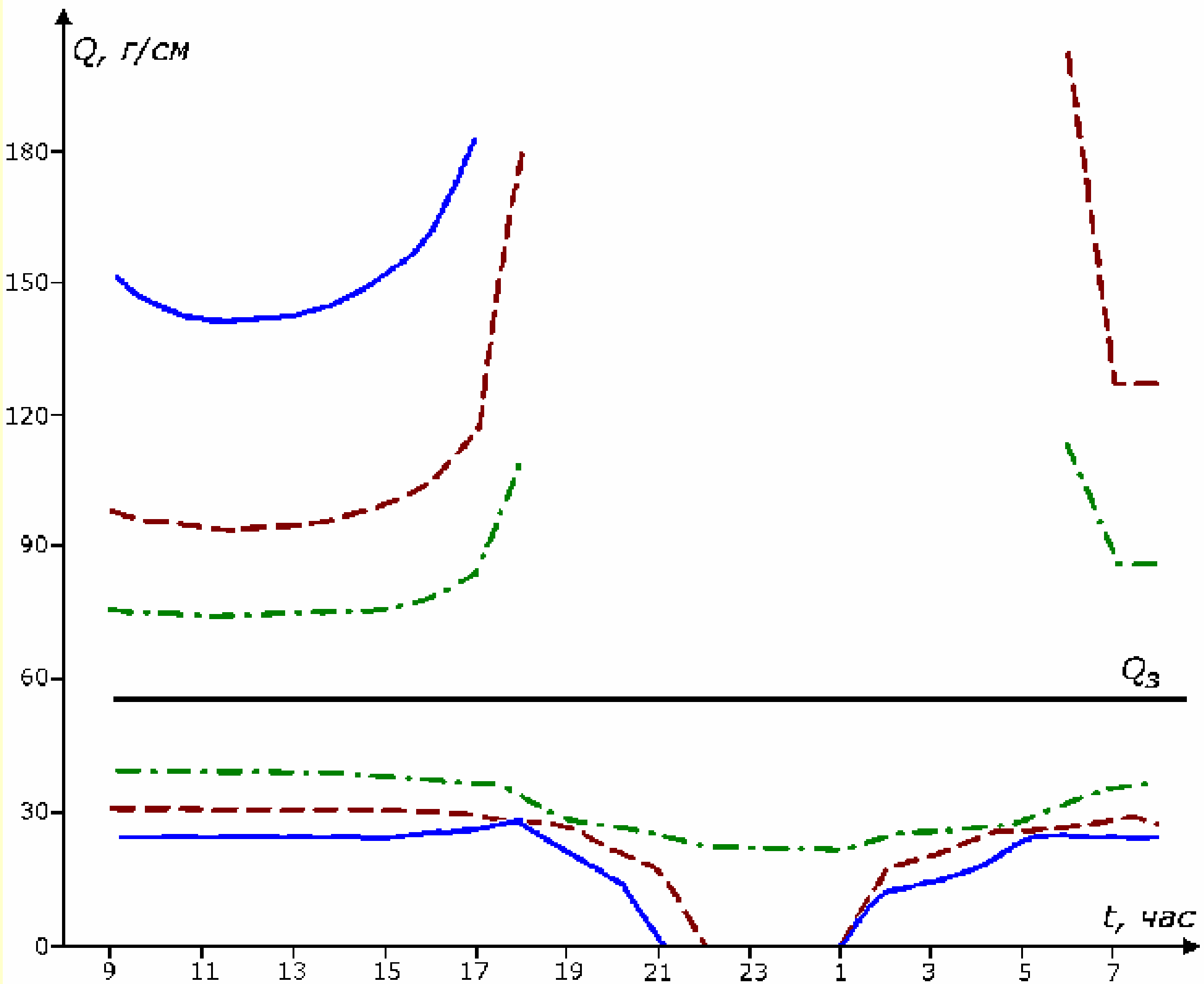


Fig. 18. Estimation of low and high bounds of summary power by rout observing data : ——— - 2 km,

— — — - 3 km, — · — · — - 5 km.

Conclusion

– Numerical analyze of monitoring data shows existence of quite simple regularities of gas and aerosol territory pollution formation

– Possibility of construction of quality models of long-term aerosol territory pollution by different types sources using small number measured points is shown. Estimations of summary emission are obtained using these models.

– Snow cover monitoring is very efficiency for control of emissions and pollution levels near enterprises.

– Using procedures of optimal planning of observation system lets essentially arise accuracy of estimation parameters and pollution fields.

– Performed results are the base for working-out of Complex monitoring system of local and region territory pollution.